

## Junior problems

J373. Let  $a, b, c$  be real numbers greater than  $-1$ . Prove that

$$(a^2 + b^2 + 2)(b^2 + c^2 + 2)(c^2 + a^2 + 2) \geq (a + 1)^2(b + 1)^2(c + 1)^2.$$

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From Cauchy-Schwarz, we know that

$$a^2 + b^2 + 2 = a^2 + 1 + b^2 + 1 \geq \frac{(a + 1)^2}{2} + \frac{(b + 1)^2}{2}.$$

By AM-GM, this is at least  $(a + 1) \cdot (b + 1)$ . Thus

$$\prod_{cyc} (a^2 + b^2 + 2) \geq \prod_{cyc} (a + 1) \cdot (b + 1) = (a + 1)^2(b + 1)^2(c + 1)^2.$$

Equality holds if and only if  $a = b = c = 1$ .

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